# A RELATIONAL THEORY OF INTERACTION AND LEARNING BASED ON CLASSIFIER SYSTEM

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## ABSTRACT

In engineering, the word interaction and learning can cover many aspects of processes of intelligent systems. In this paper, we regard the process of the interaction as transition of the status of an interface between them. From this point of view, we give a formal model of interaction and learning based on a learning system called classifier system. Firstly, we show a generall view of interaction by focusing on the role of interface. Then, the role of interface is formulated as function using the idea of relational theory of meaning in Situation Semantics. By applying the formulation, the interaction between the classifier system and the environment is defined formally in a mathematical framework of information flow called Channel Theory. In accordance with this formulation, we present an alternative view of the concept of learning, that is, learning is a process of defining a function of interface between the learning system and the environment.

#### INTRODUCTION

The word *interaction* and *learning* can cover many aspects of processes of intelligent systems. In engineering, the interaction has a mission to complete: we need to make or change intelligent systems to do certain jobs in an environment. Therefore, the changes caused by the interaction cannot be anything but something effective to the mission. When we successfully make the intelligent systems to do the jobs in the environment, these processes of interaction can be seen as a general view of the learning. There can be two different points of view to look at the change in the process. One is focusing on the status of intelligent systems or the environment, and the other is focusing on the relation between them. In the former view, the process of interaction can be regarded as a transition of internal status of intelligent systems and the environment. In the latter view, the process can be regarded as a transition of a relation between them. The latter is our standpoint in this paper and we regard the process of the interaction as transition of the status of a *interface* between them.

From this point of view, we give a formal model of interaction and learning based on a learning system called classifier system (CFS)[1]. Firstly, we show a generall view of interaction by focusing on the role of interface. Then, the role of interface is formulated as function using the idea of relational theory of meaning in Situation Semantics[2]. By applying the formulation, the interaction between the classifier system and the environment is defined formally in a mathematical framework of information flow called Channel Theory[3]. In accordance with this formulation, we present an alternative view of the concept of learning, that is, learning is a process of defining a *function* of interface between the learning system and the environment.

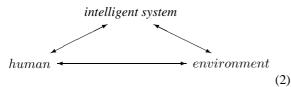
## TRANSITION OF INTERACTION

Generally speaking, when we interact with an environment, we first make approach to them and change some parts of them using our knowledge. Consequently, we know more about the environment, and we come to feel familiar with them. We need to examine the environment carefully and apply, or rather, externalize our knowledge to make them effective to a mission, while sometimes we are forced to accept them as it is. When we successfully do the job in the environment, the interaction leads our confrontation with the environment to consistency. This dialectic is sometimes referred as the movements from thesis to antithesis, and to synthesis.

Among the interactions in engineering environment, the fundamental one is a human-environment interaction.

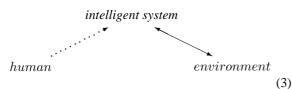
$$human \longleftrightarrow environment$$
 (1)

In this stage of interaction, the human is confronted with the environment and can hardly control it. This interaction is divided into human-intelligent system and intelligent system-environment interactions by placing an intelligent system between them.



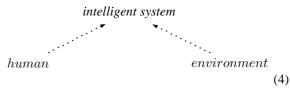
This triangle is a basic scheme of interactions in intelligent system environment where the intelligent system itself is an interface in the human-environment interaction.

By following Simon's view [4] of considering an intelligent system as an interface between its inner and outer environments, a human is included in the inner environment of the intelligent system. Therefore, the scheme turns into the following, where the arrow "



To be included in the inner environment, the human has to be familiar enough with the intelligent system, in other words, the intelligent system(interface) has to be regarded as the externalization of knowledge of the human.

On the other hand, the environment remains unknown to the human-intelligent system at this stage. When the intelligent system has the learning ability, it can increase the knowledge of the environment through the interaction. At a certain stage when it has enough knowledge to use and control the environment, it can be regarded that it virtually includes its outer environment in its own inner environment. The scheme now looks like the following.



On this stage, we successfully make the intelligent system to do the job in the environment, and the interaction leads our confrontation with the environment to consistency.

All of this amounts to saying that the process of an interaction can be seen as the transition of the state of the interface from (1) to (4). To put it another way, the role of the interface in the process is to prompt the transition. By formulating the function of an interface based on Situation Theory, we can construct a mathematical model of interactions.

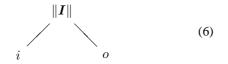
## A RELATIONAL THEORY OF FUNCTION OF IN-TERFACE

Situation Theory[2] is a mathematical theory of meaning to clarify problems in the study of information. In Situation Theory, the world is viewed as a collection of objects, sets of objects, properties, and relations. Infons are discrete items of information and real situations are objects which describe parts of the real world, i.e., real environment[7]. Infons[8] are denoted as  $\ll R, a_1, \ldots, a_i; i \gg$  where R is an n-place relation,  $a_1, \ldots, a_i$  are objects appropriate for the respective argument places of R, and i is the polarity (0 or 1). If i = 1(resp. i = 0) then the informational item that stand (resp. do not stand) in the relation R is denoted. Abstract situations are also proposed in [8] and an abstract situation is defined as a (possibly non-well-founded) set of infons. The *meaning*  $\|\Phi\|$  of an assertive sentence  $\Phi$  is a relation between the utterance u with the described situation e and write  $u \|\Phi\|e$ . This theory is called *the relational theory* of meaning.



The role of interface – intelligent system – is formulated as function by applying the idea of relational theory of meaning[9]. The inner environment of the interface, i.e., human and the outer environment of the interface are two situations, that is, parts of the real world, and the function of the interface is formulated as a pair of the inner and the outer environment in the same way as the meaning of an expression in Situation Theory is formulated as a pair of the utterance situation and the described situation. The function ||I|| of the interface I is a relation between the inner environment i with the outer environment o of the interface and write i||I||o. Following the

Situation Semantics, this formulation is called *the relational theory of function*.



# A FORMAL ANALYSIS OF INTERACTION AND LEARNING

By applying the relational theory of function, the interaction between classifier system and environment is defined formally in a mathematical framework of information flow called Channel Theory. Channel Theory is a mathematical theory to give an account of the flow of information. Classification is a mathematical structure for classifying objects introduced in [3].

**Definition 1.** We say a triple  $A = \langle \text{tok}(A), \text{typ}(A), \models_A \rangle$  is a *classification* if tok(A) and typ(A) are sets,  $\models_A$  is a binary relation between tok(A) and typ(A). We call tok(A) and typ(A) the sets of *tokens* and *types* of A respectively.

**Definition 2.** Given classifications A and B, the sum A+B of A and B is the classification defined as follows:

- 1. The set tok(A + B) is the Cartesian product of tok(A) and tok(B). The tokens of A + B are pairs  $\langle a, b \rangle$  of tokens,  $a \in tok(A)$  and  $b \in tok(B)$ .
- 2. The set  $\operatorname{typ}(\boldsymbol{A} + \boldsymbol{B})$  is the disjoint union of  $\operatorname{typ}(\boldsymbol{A})$ and  $\operatorname{typ}(\boldsymbol{B})$ . The types of  $\boldsymbol{A} + \boldsymbol{B}$  are pairs  $\langle i, \alpha \rangle$ , where i = 0 and  $\alpha \in \operatorname{typ}(\boldsymbol{A})$  or i = 1 and  $\beta \in \operatorname{typ}(\boldsymbol{B})$ .
- 3. The classification relation  $\models_{A+B}$  of A + B is defined by

$$\langle a, b \rangle \models_{\boldsymbol{A}+\boldsymbol{B}} \langle 0, \alpha \rangle \Leftrightarrow a \models_{\boldsymbol{A}} \alpha,$$
$$\langle a, b \rangle \models_{\boldsymbol{A}+\boldsymbol{B}} \langle 1, \beta \rangle \Leftrightarrow b \models_{\boldsymbol{B}} \beta.$$

Information flows from one classification to the other along with an infomorphism.

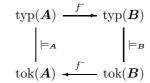
**Definition 3.** Let A and B are classifications and  $f^{\hat{}}$ : typ $(A) \rightarrow$  typ(B) and  $f^{\hat{}}$ : tok $(B) \rightarrow$  tok(A) are maps.  $f = \langle f^{\hat{}}, f^{\hat{}} \rangle$  is said to be an *infomorphism* and write  $f : A \rightleftharpoons B$  if  $f^{\hat{}}$  and  $f^{\hat{}}$  satisfies the *fundamental property* of infomorphism:

$$f^{\sim}(b) \models_A \alpha \iff b \models_B f^{\sim}(\alpha)$$

for any  $\alpha \in \operatorname{typ}(A)$  and  $b \in \operatorname{tok}(B)$ .

With two classification diagrams, infomorphisms can be depicted as follows. The notion of an infomorphism  $f : A \rightleftharpoons B$  gives a mathematical model of the

whole-part relationship, i.e., a whole modeled by a classification B and that of a part modeled by a classification A.



**Definition 4.** Let A and B are classifications. There are *natural infomorphisms*  $\varphi_A : A \rightleftharpoons A + B$  and  $\varphi_B : B \rightleftharpoons A + B$  defined as follows:

- 1.  $\varphi_{\boldsymbol{A}}(\alpha) = \langle 0, \alpha \rangle$  for each  $\alpha \in \operatorname{typ}(\boldsymbol{A})$ ,
- 2.  $\varphi_{\boldsymbol{A}}(\beta) = \langle 1, \beta \rangle$  for each  $\beta \in \operatorname{typ}(\boldsymbol{B})$ , and
- 3. for each pair  $\langle a, b \rangle \in \text{tok}(\boldsymbol{A} + \boldsymbol{B}), \varphi_{\boldsymbol{A}}(\langle a, b \rangle) = a$ and  $\varphi_{\boldsymbol{B}}(\langle a, b \rangle) = b$ .

In Channel Theory, a binary channel is a connection between two classifications. We can say that an infomorphism is a special case of channels.

**Definition 5.** A *channel* is a set  $C = \{f : A \rightleftharpoons C, g : B \rightleftharpoons C\}$  of infomorphisms. *C* is called the core of the channel C.



Theory is considered to be a set of sentences with some kinds of notions of entailment between theories and sentence.

**Definition 6.** A *theory* is a pair  $T = \langle typ(T), \vdash_T \rangle$  of a set typ(T) and a binary relation  $\vdash_T$  on subsets of typ(T). A sequent  $\langle \Gamma, \Delta \rangle$  of subset of typ(T) is said to be *constraint* of T if  $\Gamma \vdash_T \Delta$ , and *T-consistent* if  $\Gamma \nvDash_T \Delta$ . T is *inconsistent* if there is no T-consistent sequent in  $\vdash_T$ .

A theory T is *regular* iff T satisfies the following for all types  $\alpha$  and all sets  $\Gamma, \Gamma', \Delta, \Delta'$  of types:

- **1. Weakening:** if  $\Gamma \vdash_T \Delta$  then  $\Gamma \cup \Gamma' \vdash_T \Delta \cup \Delta'$ ,
- **2. Partition:** if  $\Gamma \not\vdash_T \Delta$  then there is a partition  $\langle \Gamma', \Delta' \rangle$ with  $\langle \Gamma, \Delta \rangle \leq \langle \Gamma', \Delta' \rangle$  such that  $\Gamma' \not\vdash_T \Delta'$ .

### **CLASSIFIER SYSTEM**

The CFS is a kind of production system with general mechanisms for processing rules in parallel, for adaptive generation of new rules, and for testing the effectiveness of existing rules. The rules consist of a condition and an action. The action can make true the condition of another rule. The action of a rule can also perform actions on the environment. A CFS consist of the following four components:

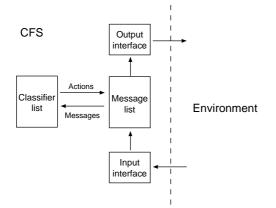


Figure. 1: Classifier System (CFS)

- message list,
- classifier list,
- input interface, and
- output interface.

The message list of the CFS plays the role of the database of the production system. A rule will be activated if the messages in the list satisfy the conditions. Activated rules may then place their actions in a new message list. The classifier list stores the rules called *classifiers*. In classifiers, the conditions are strings consisting of zeroes, ones and wildcards. These will be matched against the messages in the message list, which are ordinary bitstrings. The action part of a classifier also consists of ordinary bitstrings. The CFS takes an input from input interface, coded as a message. This message then activates classifiers which can then activate other classifier, or produce an output to the environment. In Figure 1, the main components of the CFS are shown.

The CFS and the environment can be considered to have a set of abstract situations and a set of real situations, respectively. There are two kinds of messages in the message lists of the CFS, the input messages and the output messages. The input messages are messages that are compared with information coming from the environment, and the output messages are actions sent to the environment. The input messages are used to classify the abstract situations to convert the messages into another kind of messages, that is, the output messages.

**Definition 7.** A situation theoretical classifier system  $C_s$  is a sum  $C_i + C_o$  of classifications called an *input classification* and an *output classification*, respectively.  $C_i$  and  $C_o$  are defined as follows:

- 1.  $typ(C_i)$  is a set of input messages,
- 2.  $typ(\boldsymbol{C}_o)$  is a set of output message,
- each tok(C<sub>i</sub>) and tok(C<sub>o</sub>) is a set of abstract situations,
- 4. and the classification relation  $\models_{C_i+C_o}$  is defined as

$$\langle m_i, m_o \rangle \models_{\boldsymbol{C}_i + \boldsymbol{C}_o} \langle 0, \delta \rangle \Leftrightarrow m_i \models_{\boldsymbol{C}_i} \delta,$$

 $\langle m_i, m_o \rangle \models_{C_i + C_o} \langle 1, \sigma \rangle \Leftrightarrow m_o \models_{C_o} \sigma$ for each  $m_i \in \operatorname{tok}(C_i), m_o \in \operatorname{tok}(C_o), \delta \in \operatorname{typ}(C_i)$  and  $\sigma \in \operatorname{typ}(C_i)$ .

The Environment of the CFS can be formalized in the same way that an *situation theoretical environment* classification  $E_s$  is a sum  $E_i + E_o$  of classifications called an *input environment* and an *output environment*, respectively.

**Definition 8.** An situation theoretical environment classification  $E_s$  is a sum  $E_i + E_o$  of classifications called an *input environment* and an *output environment*, respectively.  $E_i$  and  $E_o$  are defined as follows:

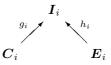
- 1.  $typ(E_i)$  is a set of input messages, i.e., messages sent to CFS through the interface,
- 2.  $typ(\mathbf{E}_o)$  is a set of output message, i.e., messages sent by the interface of CFS, and
- each tok(E<sub>i</sub>) and tok(E<sub>o</sub>) is a set of real situations, i.e, the world,
- 4. and the classification relation  $\models_{E_i+E_o}$  is defined as

 $\langle m_i, m_o \rangle \models_{\boldsymbol{E}_i + \boldsymbol{E}_o} \langle 0, \delta \rangle \Leftrightarrow m_i \models_{\boldsymbol{E}_i} \delta,$  $\langle m_i, m_o \rangle \models_{\boldsymbol{E}_i + \boldsymbol{E}_o} \langle 1, \sigma \rangle \Leftrightarrow m_o \models_{\boldsymbol{E}_o} \sigma$  for each  $m_i \in \operatorname{tok}(\boldsymbol{E}_i), m_o \in \operatorname{tok}(\boldsymbol{E}_o), \delta \in \operatorname{typ}(\boldsymbol{E}_i)$  and  $\sigma \in \operatorname{typ}(\boldsymbol{E}_i).$ 

After defining the CFS and the environment as classifications, namely  $C_s = C_i + C_o$  and  $E_s = E_i + E_o$ , the interaction between them can be considered as a flow of information between these two classifications. By following the main cycle of the learning process of classifier systems in [10], we explain how this interaction takes place in the goal state.

1. Read input messages from sensors:

For each  $i_i \in \text{tok}(I_i)$ ,  $\delta_c \in \text{typ}(C_i)$  and  $\delta_e \in \text{typ}(E_i)$ , an input interface classifier  $I_i$  conveys  $\delta_e$  to  $\delta_c$  through  $i_i$ .



2. Find classifiers that match the condition part of them:

Since there are natural infomorphisms  $\varphi_i : C_i \rightleftharpoons C_i \leftrightarrow C_i + C_o$  and  $\varphi_o : C_o \rightleftharpoons C_i + C_o$ , a type  $\delta_c$  of typ $(C_i)$  has a connection with  $\varphi_i(\delta_c) = \langle 0, \delta_c \rangle$  in typ $(C_i + C_o)$ . Then a token  $\langle m_i, m_o \rangle$  of tok $(C_i + C_o)$  such that  $\langle m_i, m_o \rangle \models_{C_i} \langle 0, \delta_c \rangle$  is found by the match procedure in the CFS.

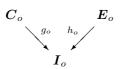
$$C_i \xrightarrow{\varphi_i} C_i + C_o \xleftarrow{\varphi_o} C_o$$

3. Fire the fireable classifiers:

As a consequence of the previous step, a type  $\sigma_c$  of typ $(C_i + C_o)$  such that  $\langle m_i, m_o \rangle \models_{C_o} \langle 1, \sigma_c \rangle$  is found and place  $\sigma_c$  in typ $(C_o)$  by the function  $\varphi_o$ .

4. Send an output message to the environment through the output interface:

For each  $i_o \in \text{tok}(\boldsymbol{I}_o)$ ,  $\sigma_c \in \text{typ}(\boldsymbol{C}_o)$  and  $\sigma_e \in \text{typ}(\boldsymbol{E}_o)$ , an output interface classifier  $\boldsymbol{I}_o$  conveys  $\sigma_c$  to  $\sigma_e$  through  $i_o$ .



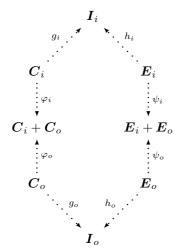
5. Iterate:

Repeat from step 1 to step 5.

The multiple interaction between them will ultimately lead to the states in which the CFS transforms the initial problematic states into goal satisfying states. In other words, when all the abstract situations in the CFS can be connected with all the real situations that are necessary to the task through the interfaces, the learning process is completed.

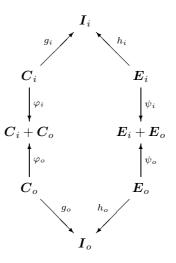
# A RELATIONAL THEORY OF INTERACTION AND LEARNING

This situation can be modeled by two interface channels between  $C_s$  and  $E_s$ , that is, when the interface channels can be defined between  $C_s$  and  $E_s$ , the learning process reached the goal states. A core of input interface channel  $I_i$  is a core of the channel between  $E_o$  and  $C_o$ , which conveys an input messages from an input environment  $E_i$  to an input classifier  $C_i$ . A core of output interface channel  $I_o$  is a core of the channel between  $E_o$  and  $C_o$ , which convey output messages from the output classification  $C_o$  to an out put environment  $E_o$ . If there are cores  $I_i$  and  $I_o$  in below diagram, the learning is completed. A diagram of a goal state CFS can be depicted as follows.



Here the arrows " $\cdots$   $\rightarrow$ " between classifications stands for a part-whole relation, i.e., infomorphism.

Given the complexity of realistic environment and the limitation of learning ability of a realistic CFS, it is generally unreasonable to assume that the CFS could be in the goal states, or rather, it is usually far from the goal states. To model such realistic states, we introduce the notion called *local logic* that was proposed in [3] to cope with logics that are both unsound and incomplete. A local logic is a triple  $\mathfrak{L} = \langle C, T, N \rangle$  where C is a classification, T is a regular theory on typ(C), and N is a subset of tok(C) called the *normal token* such that every element in N satisfies every constraints in T. Let  $\mathcal{C} = \{ f : \mathbf{A} \rightleftharpoons \mathbf{C}, g : \mathbf{B} \rightleftharpoons \mathbf{C} \}$  be a channel and  $\mathcal{L} = \langle C, T, N \rangle$  be a local logic. Then, an information  $a \models_{\boldsymbol{A}} \alpha \ (a \in \operatorname{tok}(\boldsymbol{A}) \text{ and } \alpha \in \operatorname{typ}(\boldsymbol{A})) \text{ in } \boldsymbol{A} \text{ flows to}$ **B** as  $b \models_{\mathbf{B}} \beta$  ( $b \in \operatorname{tok}(\mathbf{B})$  and  $\beta \in \operatorname{typ}(\mathbf{B})$ ) if and only if there is a token  $c \in N$  ( $N \subseteq tok(C)$ ) such that  $f(c) = a, g(c) = b, \text{ and } f(\alpha) \vdash_T g(\beta).$  By using local logic, we can describe the problematic states between realistic CFS and an environment. When a token of the input interface classification  $l_i \in tok(\mathbf{I}_i)$  is not a member of the normal token  $N_i$  of  $I_i$ , it can happen that  $l_i$  fails to connect some of the types of  $C_i$  and  $E_i$ . Consequently, the CFS cannot have sufficient information to solve a task and the system will fail to accomplish a given task in its initial problematic states. A diagram of a realistic state CFS can be depicted as follows.

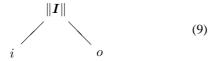


In terms of the information structure of the interfaces, the process of an interaction between the CFS and the environment can be seen as the transition of the state of the interface from (1) to (4) in the previous section, that is, part-whole relations in (3) and (4), and interactions in (1) to (3). The channel at the goal state stands for the last stage interface in (4). In addition, the interaction between a CFS and human can be seen as the same way as explained above by modeling our knowledge in the form of classification. This could lead to the alternative view of learning: learning is a process of defining functions of interface between the intelligent systems and the environment.

$$C_{s} \xrightarrow{g} E_{s}$$

$$(7)$$

To turn the relation between *i* and *o* from (7) – an initial problematic state state – into (8) – a goal state, the internal information structure of *I* has to be changed to form the part-whole relation. Recall that the *function* ||I|| of the interface *I* is a relation between the inner environment *i* with the outer environment *o* of the interface and write i||I||o. The transition from (7) to (8) is actually an activity of defining the function of *I*.



When a internal state in the interface I can successfully connect an abstract situation in human with a real situation in environment, an activity of learning process is accomplished and a function of the internal state defined as a relation between a state of human and the environment. After sufficient interactions taken place between them, from the information structural view, it can be regarded that human and the environment are virtually included in the interface, i.e., intelligent system. At this stage, human successfully make the intelligent system to do the job in the environment, and the interaction leads our confrontation with the environment to consistency.

Recent research on interactive learning systems, such as Interactive Classifier Systems (ICS) and Interactive Genetic Algorithms (IGA), could be benefited from this kind of view, since the efficiency of the interactive learning process largely depends on the structures of interfaces between the systems and human and our models proposed here could give clear indications of their formal structures.

### CONCLUSION

In this paper, we proposed a formal model of interaction and learning based on classifier system. The role of interface is formulated using the idea of relational theory of meaning in Situation Semantics. By applying the formulation, the interaction between the classifier system and the environment is defined formally in a mathematical framework of information flow called Channel Theory. In accordance with this formulation, we present an alternative view of the concept of learning, that is, learning is a process of defining a function of interface between the learning system and the environment.

#### ACKNOWLEDGMENTS

This study has been conducted in a research project 14350210 and 15700132 of Grant-in-Aid for Scientific Research from Japan Society for the Promotion of Science (JSPS).

#### REFERENCES

- [1] J. H. Holland, *Adaptation in Natural and Artificial systems*, (The University of Michigan Press, Ann Arbor, 1975).
- [2] J. Barwise and J. Perry, *Situation and Attitude*, (CSLI Publications, Stanford, 1999). (Originally published by MIT Press, Cambridge, 1983.)
- [3] J. Barwise and J. Seligman, *Information Flow: The Logic of Distributed Systems*, (Cambridge University Press, Cambridge, 1997).
- [4] H. Simon, Why should machines learn?, in R.S. Michalski, J.G. Carbonell, and T.M. Mitchell(Ed.), *Machine Learning: An Artificial Intelligence Approach*, chapter 2, (Tioga Publishing Comp., Palo Alto, CA, 1983), 25-37.
- [5] Nagasaka, I, Kikuchi, M and Kitamura, S., A Formal Analysis of Classifier System and Interface between Learning System and Environment, Proceedings of the 21st IASTED International Conference on Applied Informatics, 20-25, 2003.
- [6] Kikuchi, M. and Nagasaka, I., Situation theoretic analysis of functions for a formal theory of design, Proceedings of 14th International Conference on Engineering Design 2003, file no. 1448(CD-ROM), 2003.
- [7] Barwise, J., *Situation in Logic*, CSLI Publications, Stanford, 1988.
- [8] K. Devlin, *Logic and Information*, (Cambridge University Press, Cambridge, 1991).
- [9] Kikuchi, M. and Nagasaka, I, *Situation Theoretic Analysis of Functions for a Formal Theory of Design*, to appear in Proceedings of International Conference of Engineering Design '03.
- [10] D. E. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, (Addison-Wesley, Reading, Massachusetts, 1989).