Heuristics and metaheuristics approaches used to solve the Rural Postman Problem: A Comparative Case Study

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Abstract
The Rural Postman Problem (RPP) consists of determining a minimum cost tour of a specified arc set of a graph (G=(V,A)) with the particularity that only a subset T (T $\subseteq$ A) of arcs is required to be traversed at least once. The arcs can be directed, undirected or both.

This problem appears in a variety of practical contexts like mail, fuel and newspaper deliveries, school bus routing, electrical lines inspection, etc. (Frederickson, 1979). RPP is a NP-hard problem, therefore it has been tackled with some heuristics and metaheuristics due to the difficulty of using exact approaches to global optimality.

Up to now, and based in computational results using 26 instances described in Christofides et al. (1981) and in Corberán & Sanchis (1994), an heuristic algorithm for the RPP by Fernández de Córdoba et al. (1998) based on Monte Carlo method had obtained the best results. Baldoquín et al. (2002) developed a hybrid approach based on GRASP and Genetic Algorithm comparable with the above method testing the same instances.

In this paper a new hybrid approach based on Simulated Annealing, GRASP and Genetic Algorithm is introduced to solve the Undirected Rural Postman Problem (URPP).

We describe the design of a computational experiment to compare the performance of Monte Carlo and the two hybrid method mentioned before, using the same 26 instances. Computational results indicate that the new hybrid approach presented in this paper, and considering the instances tested, outperformed the other methods.

Key words: heuristics, metaheuristics, routing problems, GRASP, Genetic Algorithms, Simulated Annealing.

1. Introduction

The Rural Postman Problem (RPP) consists of determining a minimum cost tour of a specified arc set of a graph (G=(V,A)) with the particularity that only a subset T (T $\subseteq$ A) of arcs is required to be traversed at least once. The arcs can be directed, undirected or both. The first formulation of this problem is due to Orloff (Orloff, 1974).

RPP is a NP-hard problem (Lenstra, et. al, 1976) therefore it has been tackled with some heuristics and metaheuristics due to the difficulty of using exact approaches to global optimality.

Christofides et al. (1981) developed a heuristic algorithm using the Shortest Spanning Tree problem over a graph whose vertices correspond to the connected components of the graph induced by the edges in T. Also Christofides et al. (1981) proposed an exact algorithm with a branch and bound scheme with lowers bounds based on Lagrangean Relaxation and with upper bounds from the heuristic procedure mentioned before. 24 randomly generated problems associated to graphs up to 84 vertices, 180 edges and 74 required edges were solved up to optimality. Corberán et al. (1994) developed a branch and cut code which performs better on the Christofides instances and also solved two additional instances derived from the street network of Albaida (Valencia, Spain). To our knowledge, these 26 are the unique RPP instances with optimal solution published.

Concerning Metaheuristics in general we can mention the papers of Kang et al. (1998) using Genetic Algorithms, Rodrígues et al. (2001) using Memetic Algorithms and Baldoquín et al. (2002) using the metaheuristics GRASP (Feo et al, 1995) and Genetic Algorithms (Goldberg, 1989). The authors are not aware of any other metaheuristic used to solve the RPP.

Fernández de Córdoba et al. (1998) proposed a heuristic algorithm for the RPP based on Monte Carlo methods.

To our knowledge, up to now, and based in computational results using the 26 instances described before, the heuristic algorithm for the RPP by

Fernández de Córdoba et al. (1998) had obtained the best results, comparable with the hybrid approach developed by Baldoquín et al. (2002).

In this paper a new approach based in the metaheuristics GRASP, Simulated Annealing and Genetic Algorithms is introduced to solve the Undirected Rural Postman Problem (URPP). In a first phase Simulated Annealing approach, with a good starting solution using GRASP, is used. In a second phase an elite population is constructed with the best solutions obtained for each temperature with the Simulated Annealing approach and a genetic algorithm is applied. We also use two simplification routines to improve the tours obtained.

The URPP problem is solved on a simplified graph G'=(V', A') where V' is the set of nodes of the subgraph induced by the required edges. A' is obtained modifying the required edges T. Firstly, adding to T an edge (u,v) for each u, v $\in$ V' with cost c_{ij} equal to the length of a shortest path between u and v. Secondly, deleting all edges (i,j) for which the cost c_{ij} is equal to $c_{ik} + c_{kj}$ for some k as well as one of two parallel edges if they have the same cost.

The paper is organized in the following way: the next section gives a brief overview of different heuristic and metaheuristics approaches to solve the RPP. Section 3 introduces a new hybrid approach to solve the URPP. The experiment designed and the experimental results are summarized in Section 4. In section 5 conclusions and future perspectives are given.

2. Brief overview of heuristics and metaheuristics for the URPP

The basis of the idea of the approach based in Monte Carlo (Fernández de Córdoba et al., 1998) is to simulate a vehicle traveling randomly over a graph. The vehicle starts on an randomly node of the graph. It moves, randomly and on the basis of certain probabilities, from a node to an adjacent node until all the required edges have been traversed and then return to the initial node. This process is repeated a specified number of times and the shortest travel is considered as the output of the algorithm. In this approach 3 simplification routines, with different combinations and order in their use, are designed and tested to improve the tour found in each iteration.

The idea on the procedure based in Memetic Algoritms consists of 13 agents organized in a ternary tree structure where each one handles two tours (the current solution and the best solution until that moment) . The key issues to a near-optimal solution are recombination and local search, keeping the organization of the tree and the diversity of the population.

In Baldoquín et al. (2002) an hybrid approach based on Genetic Algorithms and GRASP is introduced to solve the Undirected Rural Postman Problem (URPP).
The method consists of two phases. In the first phase a Genetic Algorithm is used. Some members of the initial population are generated using GRASP and others randomly. A specific crossover operator designed for this problem is used and also a family elitist approach that preserves two chromosomes from each family group of four
In the second phase an elite population is constructed with the best solutions obtained in each run of the algorithm. Then the genetic algorithm is applied with this as initial population and a unary crossover operator: a modification of the inversion operator.

3. Proposed approach

In a first phase a Simulated Annealing approach is applied with a good starting solution using GRASP. We summarize the decisions concerning with parameters of the annealing algorithm itself and problem-specific.

The initial solution is the best solution regarding n/2 GRASP solutions, where n is the number of nodes of the graph. In Baldoquín, 2002 is described how was implemented GRASP for this type of problem.

The initial temperature is tei = co/(cmedia – co) where
cmedia = average cost of m random solutions
m = number of edges of the graph
co: cost of the initial solution

This initial temperature (not a high temperature) avoids to destroy the characteristics of a good initial solution and has the advantage of saving a substantial amount of solution time.

The cooling schedule is the following:
Let nrep the number of repetitions at each temperature.
In the first iteration (for the first temperature) nrep = n.
For the following temperatures nrep = nrep + m

The temperature reduction function α is

α (t)=r°t where r° = 0.75 if t $\geq$0.3
0.8 if t < 0.3

The final temperature is tf = 0.1

Neighbourhood structure:

Let $a = (a_1,..., a_i,..., a_j,..., a_n)$ a solution. We select neighbours of a in the following way: let i a randomly break point. Then:
1. we eliminate the subtour $a_i \dots a_{i+3}$ or the subtour $a_{i-3} \dots a_i$. The obtained tour is $a_1 \dots a_{i-1}\ a_{i+4} \dots a_n$ or $a_1 \dots a_{i-4}\ a_{i+1} \dots a_n$
2. we link the nodes a_{i-1} and a_{i+4} or a_{i-4} and a_{i+1} with the shortest path between them.
3. we repair the tour obtained, if it is necessary. (Fig. 1)

1. $a_1 \dots a_{i-1}\ \ a_i\ \dots\ a_{i+3}\ \ a_{i+4} \dots a_n$ (i random)

$a_1 \dots a_{i-1}\ \ \dots a_{i+4} \dots a_n$ $\longrightarrow$ Repair (if necessary)
SP (shortest path)

2. $a_1 \dots a_{i-4}\ \ a_{i-3}\ \dots\ a_i\ \ a_{i+1} \dots a_n$

$a_1 \dots a_{i-4}\ \dots\ a_{i+1} \dots a_n$ $\longrightarrow$ Repair (if necessary)
SP (shortest path)

Fig. 1: Neighbors of a solution

In a second phase an elite population is constructed with the best solutions obtained for each temperature with the Simulated Annealing approach.
A Genetic Algorithm is applied with this as initial population and the inversion operator (Bedarahally et al., 1996), with a slight modification: the cut points are based on preserving feasibility of solutions.
Let $a = (a_1 \dots a_{i-1}\ \ a_i\ \ a_{i+1} \dots\ a_{j-1}\ \ a_j\ \ a_{j+1} \dots a_n)$ a father and i, j two cut points such that (a_{i-1}, a_i) and (a_j, a_{j+1}) satisfy:
1. are non required edges or
2.the tour passes two times or more by these edges

Then the tour $a_1 \dots a_{i-1} \dots a_j\ \ a_{j-1} \dots\ a_{i+1}\ \ a_i \dots a_{j+1} \dots a_n$ is a feasible solution of the RPP where $a_{i-1} \dots a_j$ ($a_i \dots a_{j+1}$) is the shortest path between the nodes a_{i-1} and a_j (a_i and a_{j+1}) (Fig. 2).

$a_1 \dots a_{i-1}\ \ a_i\ \ a_{i+1} \dots\ a_{j-1}\ \ a_j\ \ a_{j+1} \dots a_n$

$a_1 \dots a_{i-1}\ \dots\ a_j\ \ a_{j-1} \dots\ a_{i+1}\ \ a_i \dots a_{j+1} \dots a_n$

Shortest length path

Fig. 2: Inversion operator

The general scheme of the procedure is the following:
1.Determine a good solution with GRASP
2.Apply Simulated Annealing with the solution obtained in step 1 as initial solution.
3.Let P the set of the best solutions obtained for each temperature with SA in step 2.
Then apply GA with P as initial population and an unary operator (inversion operator)

We also avoid an early convergence in this way: if after 3 reductions of temperature the best solutions at the beginning and at the end of that period are the same we apply to this solution the inversion operator and then the search continues with the new tour obtained.

4. Designed computational experiment and computational results

This approach was applied to the 26 instances described in Christofides et al. (1981) and in Corberán & Sanchis (1994). This method was compared with the heuristics of Christofides et al. (1981), Fernández de Córdoba et al. (1998) and Baldoquín et al. (2002).
In our approach we experiment with:
Different values of α, when solutions with GRASP were obtained.
Different neighbourhoods structures
Different initial temperature, different stop criterion in both phases as well as different temperature reduction function α

The best results in the experiment were obtained with: solutions GRASP with $\alpha \leq 3$, α random and the annealing parameters described before. The number of iterations in the phase II was 30.

The Table 1 shows, for the 26 instances described in Christofides et al. (1981) and in Corberán & Sanchis (1994), the number of nodes, number of edges, number of required edges, the total cost of the optimal tour, the cost of the tour obtained by the heuristics described in Christofides et al. (1981), Monte Carlo (1998), and the cost of the tour obtained by the best version of our hybrid heuristic.

The values presented in the Table I show that, according with the quality of solutions, our hybrid approach outperformed the other methods. It reaches the optimal tour in all the instances except in P25 where it reports the best result obtained so far. The hybrid approach of Baldoquín (2002) doesn't reach the optimal tour in 5 instances, Monte Carlo heuristic in 7 instances, and Christofides heuristic in 11 instances.

Inst.	# of nodes	# of edges	# of req. edges	Opt.	Chri. Heur	M.C. Heur	Hybrid Heur	New hybrid Heur.
P01	11	13	7	76	76	76	76	76
P02	14	33	12	163	**164**	163	163	163
P03	28	57	26	102	102	102	102	102
P04	17	35	22	84	84	**86**	84	84
P05	20	35	16	129	**135**	129	129	129
P06	24	46	20	102	**107**	102	102	102
P07	23	47	24	130	130	130	130	130
P08	17	40	24	122	122	122	122	122
P09	14	26	14	83	**84**	83	83	83
P10	12	20	10	80	80	**84**	80	80
P11	9	14	7	23	23	23	23	23
P12	7	18	5	21	22	21	21	21
P13	7	10	4	38	38	38	38	38
P14	28	79	31	209	**212**	209	209	209
P15	26	37	19	445	445	445	445	445
P16	31	94	34	203	203	203	203	203
P17	19	44	17	112	**116**	112	112	112
P18	23	37	16	148	148	148	148	148
P19	33	54	29	263	**280**	263	263	263
P20	50	98	63	398	**400**	**399**	398	398
P21	49	110	67	366	**372**	**368**	**372**	366
P22	50	184	74	621	**632**	621	**636**	621
P23	50	158	78	480	480	**489**	**487**	480
P24	41	125	55	405	**411**	405	405	405
P25	102	160	99	10599	-	**10784**	**10995**	**10612**
P26	90	144	88	8629	-	**8721**	**8883**	8629

Table1: Instances and best tours obtained with 4 approaches

Table 2 shows statistical results obtained with our approach with N = 30 repetitions per test problem: worst tour (cost) obtained in 30 repetitions of the algorithm, times that the optimal tour was reached, mean, standard deviation and the percentiles Q1 and Q3.

Table 3 shows the phase where the optimal cost is reached, with N = 10 repetitions per test problem.

It may be possible we could still have better results if we use the 3 simplification routines as Fernández de Córdoba did.

Inst.	Optim	Worst Cost	Repet. optim	Mean	Median	StDev	Q1	Q3
P01	76	76	30	76	76	0.00	76	76
P02	152	156	29	152.1	152	0.73	152	152
P03	102	109	12	104.9	107	2.52	102	107
P04	84	86	21	84.6	84	0.93	84	86
P05	124	127	29	124.1	124	0.55	124	124
P06	102	107	21	102.9	102	1.62	102	104
P07	130	132	26	130.3	130	0.69	130	130
P08	122	125	23	122.3	122	0.65	122	122.2
P09	83	83	30	83	83	0.00	83	83
P10	80	84	28	80.3	80	1.01	80	80
P11	23	23	30	23	23	0.00	23	23
P12	19	19	30	19	19	0.00	19	19
P13	35	35	30	35	35	0.00	35	35
P14	202	208	25	202.8	202	1.95	202	202
P15	441	448	29	441.2	441	1.28	441	441
P16	203	214	14	204.7	205	2.31	203	205.2
P17	112	114	29	112.1	112	0.37	112	112
P18	147	158	23	149.0	147	3.81	147	148
P19	257	277	14	265.1	264.5	8.17	257	274
P20	398	404	11	399.7	400	1.58	398	400
P21	366	376	9	369.6	370	2.89	366	372
P22	621	647	5	626.1	623	6.72	622	631.5
P23	475	446	15	477.3	475.5	4.28	475	477.2
P24	405	407	21	405.3	405	0.55	405	406
P25	10599	10971	0	10760	10737	106	10680	10871
P26	8629	9115	9	8689.9	8657	90.2	8629	8709

Table 2: Statistical Results with N = 30 repetitions per test problem

Inst.	GRASP	SA	Improv.	Inst.	GRASP	SA	Improv.
P01	10			P14		7	2
P02	1	9		P15	6	4	
P03		5		P16		4	4
P04		9		P17	2	8	
P05	5	5		P18	1	7	
P06		7	1	P19		1	3
P07		9		P20		4	
P08		6	3	P21		1	3
P09	10			P22			2
P10	1	9		P23		3	5
P11	10			P24		5	3
P12	10			P25			
P13	10			P26			3

Table 3: Phase where the optimal cost is reached, with N = 10 repetitions per test problem

5. Conclusions and future directions

We apply a new approach using GRASP, Simulated Annealing and Genetic Algorithm to solve the Rural Postman Problem. The results of a designed experiment realized to compare the performance of Monte Carlo heuristic and the two hybrid methods described in this paper, using a set of RPP instances taken from the literature, indicate that the new hybrid approach presented in this paper outperformed the other methods. To our knowledge, up to now and considering these instances, Monte Carlo results had been the best.

We will continue this work using the simplification routines as in Fernández de Córdoba et al. (1998). We will implement this approach to others routing problems.

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