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Overtaking a Slower-Moving Vehicle by an Autonomous Vehicle

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ABSTRACT

In the past few years there is much research on various aspects of control of autonomous vehicles. However, it seems that the problem of overtaking a slower-moving vehicle has been somewhat neglected. This paper deals with the three-phase overtaking maneuver and with designing a smooth and ergonomic optimal lane-change trajectory on a straight road.

It is shown that the absolute shape, size and time of the first-phase trajectory do not depend on the velocity of the leading, slower-moving vehicle. Only the absolute point for initiating the diversion is affected.

The relatively simple mathematical model for each lane-change trajectory is based on minimizing the total kinetic energy during the maneuver, superimposed on a "minimum-jerk trajectory". For high enough initial velocities, (above 5 m/s) explicit formulas are obtained for the optimal distance and the optimal time of the maneuver.

By using the results of the suggested model, an autonomous vehicle, equipped with appropriate sensors, can estimate the best time and place to begin and end the overtaking and its total time and distance. This may help to make a decision whether to overtake or not.

INTRODUCTION

The purpose of this paper is to design an optimal trajectory for one vehicle under normal conditions, in order to overtake a single, slower-moving vehicle on a straight, pre-determined road. It answers the following questions: At what distance from the other vehicle should the diversion from the lane begin? How long will each lane-change maneuver take? What is an optimal trajectory during each lane-change? Where and when can the overtaking vehicle return to its lane? How long will the complete overtaking maneuver take? This may aid the system to decide whether to overtake or not. In particular, if an emergency arises such as an on-coming vehicle, this decision could be crucial.

This paper is based mainly on [9]. We first analyze the general three-phase overtaking maneuver and show that the absolute shape, size and time of the first lane-change trajectory do not depend on the velocity of the obstacle. Then we design a smooth, optimal lane-change trajectory that is also ergonomic and comfortable for the passenger. The solution to the optimization problem determines the time and distance of the trajectory. It relies on formulating a nonlinear constrained optimization problem, and using optimization software to find an approximate closed-form solution. For the sake of simplicity and generality, the model does not explicitly take into account the dynamics of the vehicle or vehicle model. Therefore it can be applied to any kind of vehicle, including possible future concepts and technologies. All the forces acting upon the vehicle are embedded into one parameter – the maximal acceleration during the maneuver. As a by-product, the designed optimal trajectory complies with the recommended standards for safety and passenger comfort.

There is a vast amount of work on collision avoidance and trajectory design for autonomous vehicles. The specific problem of lane changing maneuvers is treated in [1], [3], [4], [5], [8], [10], [11] using geometric reasoning, control theory or other methods. Some of them impose specific constraints on the dynamic variables of the vehicle or on parameters like acceleration, curvature, jerk, etc. and some specifically minimize parameters like time, distance, acceleration, curvature and such.

In [11] and [1] results are obtained for the distance to begin the diversion and the total time the lane-change maneuver takes, considering the vehicle dynamics. In [11] the objective is to minimize the clearing distance for emergency maneuvers in such a way that the diversion is still safe and feasible. Although there is some similarity between the results of [11] & [1] and the ones obtained in this paper for the lane-change trajectory, the trajectories they generate are not necessarily smooth and they do not obtain closed-form formulas. Furthermore, they only consider lane-change maneuvers and not overtaking a moving vehicle.

The presentation will outline the basic results of this research, including an animated demo.

THE GENERAL OVERTAKING MANEUVER

An autonomous vehicle *P* (passing) is driving at a velocity *V*, say in the *x* direction. In front of it another car, *O* (obstacle) is driving at a constant velocity $0 < V_1 < V$ in the same direction. Vehicle *P* intends to pass it.

An overtaking maneuver consists of three phases: (a) diverting from the original lane, (b) driving straight in the adjacent lane, (c) returning to the lane.

We shall consider planar translation, i.e. $x(t)$ and $y(t)$, where *x* is the original direction of motion and *y* is the orthogonal direction of diversion. The lane-changing trajectories could be any one of those suggested in the literature, as presented in the introduction. However, a different option is suggested in the following sections.

Phase (a): Diverting from the lane. Denote the total *x*-direction distance traveled during the lane-change maneuver by *D*, the total *y*-direction distance by *W* (the width of the lane or of the diversion) and the total time duration by *T*. These values are determined by the specific lane-change trajectory. We set the origin of the (x, y, t) system at the point on the x-axis next to the peak of the diversion, where the front of vehicle *P* is located in the adjacent lane next to the rear of vehicle *O,* and set the time at that point to zero. In other words, the rear of vehicle *O* is located at (0,0,0) at the same time (t=0) when the front of vehicle *P* is at (0,*W*,0). See Figure 2-1.

It turns out that the absolute shape, size and time of the trajectory of vehicle *P* do not depend on the velocity of vehicle *O*. Due to the nonlinear nature of the problem, it is impossible to use relative velocities, but this is not necessary since there is a simple explanation, as presented in Figure 2-1 and the caption below it.

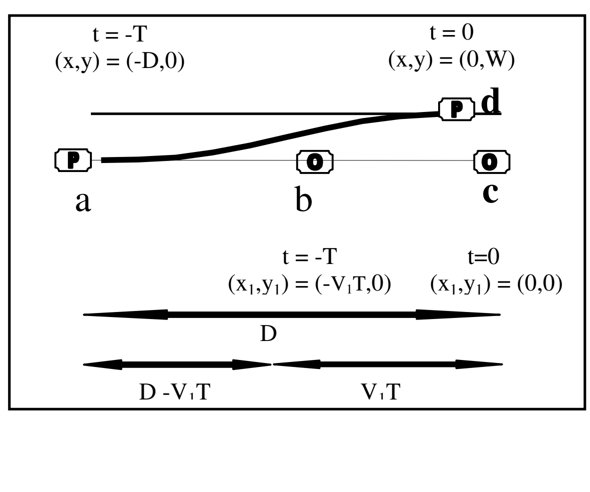

Fig. 2-1: Overtaking a moving vehicle. At time $t = -T$, vehicle *P* is located at the point **a**, and vehicle *O* is at the point **b**. After *T* seconds, at time *t=0*, vehicle *O* is located at **c** and *P* is at **d**. Vehicle P needs to travel from point **a** to point **d** during the same time interval that *O* travels from **b** to **c**. The time interval *T* and horizontal distance *D* depend only on the lane-change trajectory of vehicle *P* and not on *O*. The only role of vehicle *O* is to determine the absolute location of the point **c**, hence that of the point **a**.

To determine the point for beginning the diversion, vehicle *P* calculates *its own* *T* and *D*, based on the parameters of its own lane-changing trajectory and the width *W*. It estimates the velocity V_1 of vehicle *O,* and calculates the relative distance $D_{rel} = D - V_1 T$. At this distance from *O,* it begins the diversion. Some examples are presented in Table 3-A.

Phase (b): Driving straight in the adjacent lane. For simplicity we assume that while vehicle *P* is moving in parallel to vehicle *O*, both vehicles maintain their original velocities *V* and V_1 respectively. If the length of vehicle *P* is *L* and that of *O* is L_1, then the passing vehicle must travel a *relative* distance of at least $L + L_1$ at a relative velocity of $V - V_1$ before it can begin returning to its lane. This would take an amount of time $T_{(b)} = \frac{L + L_1}{V - V_1}$. Therefore the *absolute* distance that *P* must travel is at least $D_{(b)} = \frac{L + L_1}{V - V_1} V$. For example, if *L*=5 meters and L_1=6 meters, *V*=25m/s and V_1=20m/s, then vehicle *P* must travel at least $D_{(b)} = \frac{5+6}{25-20} \cdot 25 = 55$ meters. This would take $T_{(b)} = \frac{5+6}{25-20} = 2.2$ seconds. Of course *P* must keep a security distance before returning, since its velocity may decrease slightly during lane-change.

Phase (c): Returning to the lane. Vehicle *P* can now use the symmetric lane-change trajectory to return. We simply use symmetry and time reversal. The origin of the new space-time coordinate system is located in front of vehicle *O*, next to the rear of vehicle *P* that is traveling in the adjacent lane. In the new lane-change trajectory we simply substitute $x_{new}(t) = -x(-t)$, $y_{new}(t) = y(-t)$.

To conclude this section, if both vehicles maintain their parameters and velocities, then the total overtaking maneuver would take *at least* $2T + T_{(b)}$ seconds and the total *x*-direction distance is at least $2D + D_{(b)}$ meters. If *P* changes its velocity while in the adjacent lane (say, if an emergency situation arose), then the above formulas for $T_{(b)}$ and $D_{(b)}$ will pertain to its average velocity. If it also changes its dynamic variables such as the amount of power or torque when returning to the lane, then its trajectory is determined accordingly, hence the *T* and *D* values for returning will also be the new ones. In any case, it is possible to estimate the total time and distance of the complete overtaking maneuver.

DESIGN OF AN OPTIMAL LANE-CHANGE TRAJECTORY

We now suggest a design of an optimal lane-change trajectory for vehicle *P*. We regard vehicle *P* as a point mass[1]. The maximal resultant force acting on the vehicle, which is proportional to the norm of the maximal acceleration vector, depends on various external and internal conditions such as inclination, friction and especially the amount of power that the (human or automatic) driver chooses to exert[2]. In an emergency situation, the system may use higher acceleration, which is of course bounded by the vehicle's capabilities and the external conditions. These factors vary with different conditions, but are taken as constant during the short time interval of the maneuver. Thus the assumption is that the acceleration has a constant bound.

If the trajectory is designed for comfort, then the system should choose the acceleration bound so that the lateral acceleration does not exceed the recommended bound for passenger comfort, which is 3-4 m/s^2 (see [6] for example). However, it is stated in [12] that human drivers, under normal conditions, usually use an acceleration of about 1 m/s^2 during overtakes. By Figure 6-3 it can be seen that the lateral component of the acceleration vector is the dominant one.

Formulating the equations of motion: The suggested design of a lane-change trajectory is based on underlying polynomial equations, superimposed with minimizing the total kinetic energy of vehicle *P* during the maneuver. It is convenient to consider the maneuver for phase (c), returning to the lane.

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[1] Although we consider vehicle P as a point mass located at its CG, all distances relate to its front for phase (a) and (b) maneuvers and to its rear for phase (c) maneuvers.

[2] For a conventional human-driven vehicle this may be the pressure on the gas pedal or brakes, the gear being used, etc.

To determine the trajectory of vehicle *P*, we fit a polynomial expression for $x(t)$ and $y(t)$, satisfying appropriate boundary conditions. For simplicity, we assume the accelerations at the initial and final points of the lane-changing maneuver are both zero, and the initial and final velocities are equal. These assumptions may not be realistic, but they are a simplifying approximation. During the maneuver itself, the velocity and acceleration are not assumed to be constant.

Let *D* and *T* be as in the previous section. These values are as yet unknown, but are determined by the optimization. The known parameters of the optimization problem are: *V* = the initial and final velocity of *P*; *W* = the width of the lane or of the diversion; *A* = the magnitude of the maximal resultant acceleration of *P*. All the known and unknown parameters are positive. The boundary conditions are:

$$x(0)=0 \quad x(T)=D \quad \dot{x}(0)=\dot{x}(T)=V \quad \ddot{x}(0)=\ddot{x}(T)=0$$
$$y(0)=W \quad y(T)=0 \quad \dot{y}(0)=\dot{y}(T)=0 \quad \ddot{y}(0)=\ddot{y}(T)=0 \qquad (3.1),(3.2)$$

By writing down a general 5-th degree polynomial and applying the boundary conditions (3.1), we obtain the following equations:

$$x(t)=Vt+(VT-D)\left(-10\left(\frac{t}{T}\right)^3+15\left(\frac{t}{T}\right)^4-6\left(\frac{t}{T}\right)^5\right) \qquad (3.3)$$

$$y(t)=W+W\left(-10\left(\frac{t}{T}\right)^3+15\left(\frac{t}{T}\right)^4-6\left(\frac{t}{T}\right)^5\right) \qquad (3.4)$$

The form of these equations is well known in the context of biological motion [2], where it was shown that minimizing the total jerk yields a fifth-degree polynomial. Thus it is called a “minimal jerk” trajectory. Similar equations for the trajectory of an autonomous vehicle were suggested in [1].

FORMULATING THE OPTIMIZATION MODEL

The coefficient $(VT-D)$ in (3.3) represents an upper bound on the *additional* distance that the vehicle traveled due to the diversion. Denote this difference by $S=VT-D$. Because of the diversion and the slight reduction of velocity, *T* is a bit larger than the original time-to-collision. Therefore this difference is positive. The variable *S* is just a substitution that makes the expressions simpler. It is more convenient to use the variables *(T,S)* instead of (*T,D*), so we shall use them henceforward. But since we are actually seeking the optimal value of *D,* then we find the optimal *S** and *T** and substitute

$$D^*=VT^*-S^* \qquad (4.1)$$

The constraints of the optimization problem are as follows:

1) **The velocity constraint**: We want to insure that the motion in the *x*-direction is always forward, or that $\dot{x}(t)\geq 0$ for all $0\leq t\leq T$. From (3.3) it can be shown that $\dot{x}_{\min}=V-\frac{15S}{8T}$. Therefore we must satisfy the *velocity constraint*:

$$8VT\geq 15S \qquad (4.2)$$

2) **The acceleration constraint**: The maximal acceleration or deceleration from (3.3) and (3.4) are: $\max\sqrt{\ddot{x}^2+\ddot{y}^2}=10\frac{\sqrt{3}}{3}\frac{\sqrt{S^2+W^2}}{T^2}$. This should be equal to *A,* the norm of the maximal acceleration vector of vehicle *P,* chosen to use during the maneuver. Thus the following *acceleration constraint* must be satisfied:

$$g(T,S)=\frac{S^2+W^2}{T^4}=\frac{3}{100}A^2 \qquad (4.3)$$

The objective function to be minimized is the total kinetic energy. In terms of *x* and *y* from equations (3.3), (3.4) and omitting the constant mass coefficient, it is:

$$TKE=f(T,S)=\int_0^T(\dot{x}^2+\dot{y}^2)dt=\frac{10}{7T}\left(S^2+W^2\right)-2VS+V^2T \qquad (4.4)$$

The optimization problem we wish to solve for vehicle *P* is:

$$\begin{cases} \textit{Minimize} \quad f(T,S)=\frac{10}{7T}(S^2+W^2)-2VS+V^2T \\ \textit{Subject to} \\ 1) \quad 8VT\geq 15S \\ 2) \quad g(T,S)=\left(\frac{W^2+S^2}{T^4}\right)=\frac{3}{100}A^2 \end{cases} \qquad (4.5)$$

It is proven in [9] that problem (4.5) attains a unique global solution.

APPROXIMATING OPTIMAL *T** AND *D**

Finding an analytic solution for problem (4.5) using methods like Karush Kuhn Tucker is extremely difficult. Therefore a numerical method was used, together with the optimization software Lingo™. Running the program with various values of the parameters and seeking the optimal values of the unknown variables obtained the results. Each parameter was isolated and varied while the others were kept constant.

The resulting values for D* and *S** were plotted against each varying parameter separately. For high enough velocities, (say $V \geq\sim 5m/s$), it was obvious that the optimal value of *D** is approximately proportional to *V*, to the square root of *W*, and inversely proportional to the square root of *A*. These proportions were also obtained in [1]. The proportionality constant was found to be approximately[3] α=2.4. The nature and justification of this coefficient is given in [9]. It was also quite obvious for high enough velocities, that *S** is approximately proportional to $W^{3/2}$, to $\sqrt{A}$, and inversely proportional to *V*. The proportionality constant was found to be approximately[4] $\beta = 1.73\,(\approx \sqrt{3})$. Therefore it can be concluded for high enough values of the initial velocity *V*, that:

$$D^* \approx 2.4V\sqrt{W/A} \tag{5.1}$$

$$S^* \approx \sqrt{3}\,\frac{W^{3/2}\sqrt{A}}{V} \tag{5.2}$$

By substituting $T^* = \frac{S^* + D^*}{V} = \frac{S^*}{V} + \frac{D^*}{V}$ from (4.1), we obtain:

$$T^* \approx \sqrt{3}\,\frac{W^{3/2}\sqrt{A}}{V^2} + 2.4\frac{\sqrt{W}}{\sqrt{A}} \tag{5.3}$$

These formulas were validated against the results obtained from the optimization software with many test runs, and gave a highly accurate approximation. There were relatively significant deviations only for cases of very low velocities.

Some examples of the optimization are presented in Table 5-A, as well as the relative distances for beginning the diversion in phase (a). The units in the table are meters, seconds etc.

V	W	A	D^*	T^*	V_1	D_{rel}
15	3	3	36	2.47	12	6.36
25	3	4	52	2.1	15	20.38
25	4	2	84.96	3.43	20	16.38
35	3.5	4	78.67	2.26	20	33.35

Table 5-A: Some examples of optimal distances, relative distances and time for a lane-change maneuver.

It is also shown in [9] that the optimal time of the lane-change is bounded from above and below regardless of the initial velocity. Figure 5-B shows the trajectory generated by the model.

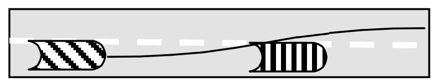


Figure 5-B: Optimal lane-change trajectory generated by model (4.5).

VELOCITY, CURVATURE, ACCELERATION AND JERK PROFILES

This section analyzes various aspects of the optimal lane-change trajectory. The figures represent examples of velocity, acceleration, jerk and curvature profiles as determined for an optimal trajectory generated by the model. The time scale in the figures is the relative time, and all units are in meters and seconds.

1. First, we note that the **velocity profile** in Figure 6-1 is lower bell-shaped and that the velocity decreases (typically by about 0.5%-1.2%) during lane-change.

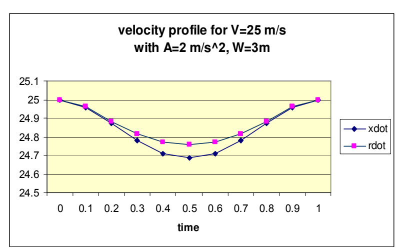
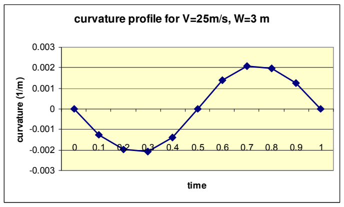


Fig. 6-1: Lower bell shaped velocity profile.

2. Now we look at the **curvature.** For safety in conventional vehicles, the radius of curvature must be bounded from below, depending on the speed of travel.

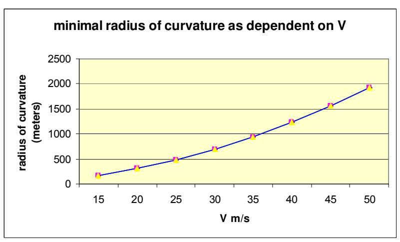
[3] n=50, ave=2.4041, SD=0.0008
[4] n=50, ave=1.7268, SD=0.027

Figure 6-2(a) shows the curvature profile for velocity of 25 m/s with lane width of 3 meters. Since the maximal curvature in this case is approximately 0.002 m^{-1}, then the minimal radius of curvature is about 500 meters. This complies with the standard recommendation of $\rho \geq 470m$ for this velocity, given in [8].



6-2(a)

We also want to see how the maximal curvature and the minimal radius of curvature during the maneuver, depend on the velocity parameter *V*. Figure 6-2(b) is the minimal radius of curvature plotted against the velocity.



(b)

Figure 6-2: (a) Curvature profile over the time of the lane-change maneuver.

(b) Minimal radius of curvature as dependent on V.

3. As for **acceleration**, we wish to compare the lateral and the longitudinal components of the acceleration profiles along the trajectory, which are the acceleration components that are respectively normal and tangent to the path.

Figure 6-3 is a comparison between lateral and longitudinal accelerations for two different cases. It shows that the lateral component of acceleration is the dominant one. This kind of profile is in accordance with the "desired shape of lateral acceleration", as specified in [4].

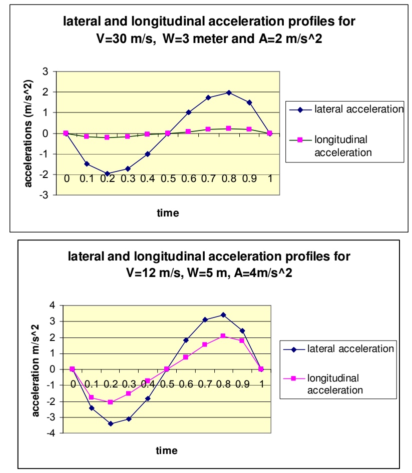
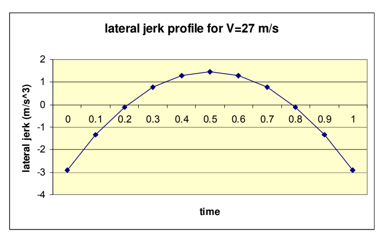
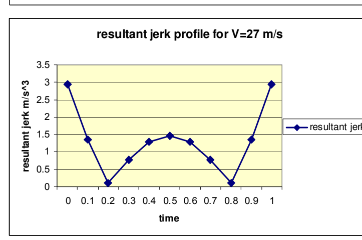


Figure 6-3: Comparisons of lateral and longitudinal acceleration profiles, using two different situations.

4. Now we consider the **jerk** (third time-derivative of the position vector) during the maneuver. The recommended maximal lateral jerk (in absolute value) for passenger comfort should not exceed 2.4 g/s [1] (~0.24 m/s^2) and for passenger safety, the resultant jerk should not exceed 5 m/s^3 [7]. Figure 6-4(a) shows the lateral jerk profile for initial velocity of 27 m/s, acceleration bound of 2 m/s^2 and diversion of 3 meters. Although the lateral jerk at the beginning and end of the maneuver is about 2 m/s^2 which is 25% higher than the recommended value, this is only an instantaneous discomfort. During most the maneuver, the bound for comfort is satisfied. Figure 6-4(b) shows a profile of the resultant jerk under the same conditions. The figure shows that the recommended bound for passenger safety is not violated.



(a)



(b)

Figure 6-4: Jerk profiles. (a) Lateral jerk. (b) Resultant jerk.

CONCLUSIONS

This paper deals with overtaking a slower-moving vehicle on a straight road. The first part analyses the three-phase maneuver and shows that the velocity of the obstacle does not affect the absolute shape or time of the first lane-change trajectory. For the straight part, the minimal time and distance traveled before returning to the lane, is obtained. These values do depend on the other vehicle's velocity.

The second part of the paper designs a smooth and comfortable optimal lane-change trajectory. The optimization objective is to minimize the total kinetic energy of the passing vehicle during the lane-changing maneuver, superimposed on a "minimal-jerk" trajectory. Two constraints are imposed, one to guarantee that the motion of the overtaking vehicle is always forward, and one to comply with the maximal acceleration chosen to use during the maneuver.

The solution of the optimization problem determines the optimal time and distance of the lane-change maneuver. These values in turn, determine the trajectory itself. For high enough initial velocities, explicit closed-form formulas were developed, which approximate the optimal values of these parameters. The formulas were compared with results obtained from the optimization software and showed high accuracy.

An autonomous vehicle equipped with appropriate sensors and programmed with these formulas, can calculate its trajectory and the best place to begin and end the maneuver.

Future research on this subject includes: 1. Checking whether human drivers use a similar paradigm as the one suggested in this paper. 2. Designing a trajectory for overtaking on a curved road. 3. Formulating the problem for low velocities (below 5 m/s).

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